Structure refinement using molecular dynamics simulations (NMR observables)

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105.0

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¹H (ppm)





Simulate with restraints

- Bias the coordinates **r** to make <<u>Q</u>(**r**)>_{sim} approach <u>Q</u>^{exp}
 - Add an extra potential energy term to the force field

 $V^{tot}(\mathbf{r}) = V^{phys}(\mathbf{r}) + V^{restr}(\mathbf{r})$

Functional form

- Full or half harmonic at short range

$$V_{harm}^{restr}(\mathbf{r}) = \frac{1}{2} K^{Qr} \left[\left\langle Q(\mathbf{r}) \right\rangle - Q^{\exp} \right]^2 \quad for \left\langle Q(r) \right\rangle < Q^{\exp} + \Delta Q$$

Bounded gradient (force) at long range

$$V_{lin}^{restr}(\mathbf{r}) = K^{Qr} \left[\left\langle Q(\mathbf{r}) \right\rangle - Q^{\exp} - \frac{1}{2} \Delta Q \right] \Delta Q \quad for \left\langle Q(r) \right\rangle \ge Q^{\exp} + \Delta Q$$

- Continuous, continuous derivative

Include

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- averaging over time (instantaneous vs. time-averaged)
- averaging over molecules if Q depends on multiple molecules
- enhanced sampling techniques if r(Q) is multi-valued

The structure (ensemble) determination problem

Examples of (*observable*) quantities Q(r):

NOE intensities or distances $\langle I_{i,j} \rangle \sim \langle r_{ij}^{-p} \rangle$ with p = 3 or 6³J-coupling constants $\langle {}^{3}J_{i,j} \rangle = a \langle \cos^{2} \theta_{i,j} \rangle + b \langle \cos \theta_{i,j} \rangle + c$ Residual dipolar couplings $\langle D_{i,j} \rangle = a \langle \cos^{2} \theta_{ij} \rangle + b$ Chemical shifts $\langle \sigma \rangle$ Structure factors (amplitudes) $\langle F_{hkl} \rangle$ FRET Efficiencies $\langle E(R, \theta) \rangle$ CD spectra $\langle I(\lambda) \rangle$



Atom-atom distance restraining

Distance restraint:

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 d_0 involving atoms *i* and *j*, derived from experiment

d(t) actual distance at time t $d(t) = \sqrt{\left[\mathbf{r}_{i}(t) - \mathbf{r}_{i}(t)\right]^{2}}$

Instantaneous distance restraint:

$$V^{dr} = 0 if d(t) < d_0$$

$$V^{dr} = \frac{1}{2} K^{dr} \left[d(t) - d_0 \right]^2 if d(t) > d_0$$

$$f_{ix} = 0 if d(t) < d_0$$

$$f_{ic} = -K^{dr} \left[d(t) - d_0 \right] \frac{x_{ij}(t)}{r_{ij}(t)} if d(t) > d_0$$

Atom-atom distance restraining

• Include time averaging

$$\overline{d}(t) = \left[\frac{1}{t}\int_{0}^{t} d(t')^{-3} dt'\right]^{-1/3} \qquad \overline{d}(t) = \left[\frac{1}{N_{t}}\sum_{n=1}^{N_{t}} d(t_{n})^{-3}\right]^{-1/3}$$

Use 1/3 averaging because <I^{NOE}> is proportional to <d⁻³>

Time-average restraint:

• Force becomes smaller with growing N_t, so damp the memory

• Time-averaging with a memory relaxation time τ :

 $\overline{d}(t) = \left[\frac{1}{t}\frac{1}{1-e^{-t^{1/\tau}}}\int_{0}^{t}e^{-(t-t^{\prime})/\tau}d(t-t^{\prime})^{-3}dt^{\prime}\right]^{-1/3}$





Instantaneous restraints

Applying *instantaneous* distance restraints

vnamics

hermod

S

lied Statisti

puts an extra force on the atoms which pulls them to the experimental NOE distance

Atoms do get close, but quite a lot of strain is present in the system

Small distance fluctuations Yet, *small* bound violations remain present





Time-averaged restraints

- Time-averaged distance restraints
- Extra forces on the atoms to enforce that the NOE distance is fulfilled on average
- Tyr13 is flipping back and forth



Large distance fluctuations Yet, *no* bound violations are present







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The structure (ensemble) determination problem

Examples of (*observable*) quantities *Q*(**r**):

- NOE intensities or
distances $\langle I_{i,j} \rangle \sim \langle r_{ij}^{-p} \rangle$ with p = 3 or 6**3J-coupling constants** $\langle {}^{3}J_{i,j} \rangle = a \langle \cos^{2} \theta_{i,j} \rangle + b \langle \cos \theta_{i,j} \rangle + c$ Residual dipolar couplings $\langle D_{i,j} \rangle = a \langle \cos^{2} \theta_{ij} \rangle + b$ Chemical shifts $\langle \sigma \rangle$
- Structure factors (amplitudes) $\langle F_{hkl} \rangle$
- CD spectra $\langle I(\lambda) \rangle$





Applying restraints

Instantaneous J-value restraining

$$V^{Jr} = \frac{1}{2} K^{Jr} \left[J(\phi(\mathbf{r})) - J_0 \right]^2$$

• Time-average J-value restraining

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$$V^{Jr} = \frac{1}{2} K^{Jr} \left[\overline{J}(\phi(\mathbf{r})) - J_0 \right]^2$$



Problem with time averaging: The force remains if the instantaneous value is already at the target For distances: -Van der Waals repulsion counteracts it -Only attractive forces used

-r⁻³ averaging favours short distances

For J-values: time-averaging leads to large fluctuations

Double harmonic J-value restraining

$$V^{Jr} = \frac{1}{2} K^{Jr}_{double} \left[J(\phi(\mathbf{r})) - J_0 \right]^2 \left[\overline{J}(\phi(\mathbf{r})) - J_0 \right]^2$$













Application: ³J-value biasing

Average ³J-coupling constants

$$\bar{J}(\mathbf{r}) = \frac{1}{\tau} \frac{1}{1 - e^{-t/\tau}} \int_{0}^{t} e^{-(t-t')/\tau} J(t') dt$$

τ - memory decay time

• Restraining potential is the sum of N_{le} terms

$$V^{Jr}(\phi(\mathbf{r})) = \sum_{i=1}^{N_{le}} V_i^{le}(\phi)$$

which are Gaussian Local Elevation functions, with an extra factor

$$V_i^{le}(\phi) = k^{Jr} W_{\phi i}(t) e^{-(\phi - \phi_i^0)^2 / 2(\Delta \phi^0)^2}$$

a possible potential V_i at equally separated values of ϕ_i , distance $\Delta \phi^0$ $N(\phi = \phi_i^0)$ in original local elevation formulation replaced by $w_{\phi i}(t)$











Are the NMR model structures representative for the real conformational ensemble ? \rightarrow improve the protocol for structure determination by including time averaging

Are there inconsistencies between NOEs and ³J values ?

 \rightarrow find a conformational Boltzmann ensemble that satisfies all the primary data

Unrestrained molecular dynamics simulations



MD with distance restraints

- 1. **instantaneous distance restraints**: put an extra force on the atoms pulling them instantaneously to the experimental NOE distance \rightarrow strain in the system
- time-averaged distance restraints: put an extra force on the atoms such that the NOE distance is fulfilled on average → side chains flipping back and forth





with one structure



MD simulations with time-averaged NOE distance restraints and instantaneous restraining on ³J-values or local elevation biasing on ³J-values



Using time-averaged distance restraints based on NOEs and local elevation biasing on ³J-coupling constants an ensemble of structures was obtained that satisfies the entire set of experimental data

The results are force field independent







The structure (ensemble) determination problem

Examples of (*observable*) quantities Q(r):

• NOE intensities or distances $\langle I_{i,j} \rangle \sim \langle r_{ij}^{-p} \rangle$ with p = 3 or 6

 $\langle \sigma \rangle$

 $\langle I(\lambda) \rangle$

• ³J-coupling constants

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- $\left<^{3}J_{i,j}\right> = a\left<\cos^{2}\theta_{i,j}\right> + b\left<\cos\theta_{i,j}\right> + c$ $\left< D_{i,j}\right> = a\left<\cos^{2}\theta_{ij}\right> + b$
- Residual dipolar couplings
- Chemical shifts
- Structure factors (amplitudes) $\langle F_{hkl}
 angle$
- FRET Efficiencies $\langle E(R,\theta) \rangle$
- CD spectra
 - (implemented in GROMOS)





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PSEUDOATOM AND MULTIPLICITY CORRECTIONS

<text><text><text><list-item><list-item><list-item>**Determine the position on the fly from the relevant heavy atoms transfer the position on the fly from the relevant heavy atoms transfer the forces to the heavy atoms 1 The signals of multiple protons may not be distinguishable 9 The signals of multiple protons may not be distinguishable 9 The signals of multiple protons may not be distinguishable 9 Given the experimental settings 1 Apply the distance restraint to a pseudo atom 1 calculate the position on the fly from the relevant heavy atoms transfer the forces to the heavy atoms**



Pseudo atom corrections

- Identical spins are considered to be represented by one pseudo-atom.
- A multiplicity correction for the fact that the real signal comes from more than one atom is added
- A pseudoatom correction for the fact that the 'closest' of the atoms will contribute most, while the pseudo atom is further away.

Corrections

	Wüth	Wüthrich (1983)		er (1996)	GROMOS (1996)
		corr (Å)	Z	corr (Å)	corr (Å)
3.	H H	+1.0	×2 ^{1/p}	+0.7	+0.9
5.	H H H	+1.5	×3 ^{1/p}	+0.4	+1.0
6.	H H H H	+2.9	×6 ^{1/p}	+1.5	+2.2
7.	H H	+2.0	×2 ^{1/p}	+2.0	+2.1