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Institute of molecular modeling and simulation

Calculation of free energy from molecular simulations



Schedule

Time	Mon. 20/11/17	Tue 21/11/17	Wed. 22/11/17	Thur. 23/11/17	Fri. 24/11/17	Mon. 27/11/17	Tue 28/11/17	Wed. 29/11/17	Thur. 30/11/17	Fri. 01/12/17
9:00	Free Time	Welcome <small>JAG</small>	Free Time	Free Time	Free Time	Free Time	Free Time	Free Time	Free Time	Free Time
9:30		Lecture 1: Overview and Introduction <small>JAG</small>	Lecture 4: Thermodynamics <small>JAG</small>	Lecture 7: Classical Mechanics I <small>JAG</small>	Lecture 10: Ensembles I <small>CO</small>	Lecture 13: Free energies: reaction coordinates <small>JAG</small>	Lecture 16: Calculating properties from simulations <small>JAG</small>	Lecture 19: Electrostatics <small>WvG</small>	Lecture 22: On the ethics of the academic endeavour: where do we go? <small>WvG</small>	Lecture 24: Left-overs/questions and future perspectives <small>WvG</small>
10:15		Break <small>JAG</small>	Break <small>JAG</small>	Break <small>JAG</small>	Break <small>CO</small>	Break <small>JAG</small>	Break <small>JAG</small>	Break <small>WvG</small>	Break <small>WvG</small>	Break <small>WvG</small>
10:30		Lecture 2: Molecular Simulations MD/SD/MC <small>CO</small>	Lecture 5: Force-Field Development <small>CO</small>	Lecture 8: Classical Mechanics II <small>JAG</small>	Lecture 11: Ensembles II <small>JAG</small>	Lecture 14: Boundary Conditions I <small>WvG</small>	Lecture 17: Comparison with Experiments <small>WvG</small>	Lecture 20: Polarization <small>WvG</small>	Lecture 23: QM/MM <small>WvG</small>	Lecture 25: Students plans I <small>WvG</small>
11:15		Coffee Break <small>CO</small>	Coffee Break <small>CO</small>	Coffee Break <small>CO</small>	Coffee Break <small>CO</small>	Coffee Break <small>WvG</small>	Coffee Break <small>WvG</small>	Coffee Break <small>WvG</small>	Coffee Break <small>WvG</small>	Coffee Break <small>WvG</small>
11:45		Lecture 3: How to simulate using GROMOS <small>CO</small>	Lecture 6: Structure Refinement <small>CO</small>	Lecture 9: Analyzing with GROMOS <small>CO</small>	Lecture 12: Free energies: alchemy <small>CO</small>	Lecture 15: Boundary Conditions II <small>WvG</small>	Lecture 18: Searching & Enhanced Sampling <small>WvG</small>	Lecture 21: Multi-resolution simulations <small>WvG</small>	Lecture 23: QM/MM <small>WvG</small>	Lecture 26: Students plans II <small>WvG</small>
12:30		Break for lunch, self-study, discussion.* <small>CO</small>	Break for lunch, self-study, discussion.* <small>CO</small>	Break for lunch, self-study, discussion.* <small>CO</small>	Break for lunch, self-study, discussion.* <small>CO</small>	Break for lunch, self-study, discussion.* <small>WvG</small>	Break for lunch, self-study, discussion.* <small>WvG</small>	Break for lunch, self-study, discussion.* <small>WvG</small>	Break for lunch, self-study, discussion.* <small>WvG</small>	Tutorial 10: **
14:00	Registration	Tutorial 2: Running MD GROMOS Tutorial	Tutorial 3: Running MD Students Plans	Tutorial 4: Running MD Students Plans	Tutorial 5: Running MD Students Plans	Tutorial 6: Statistical Mechanics exercises	Tutorial 7: Analyzing MD: GROMOS Tutorial	Tutorial 8: Analyzing MD: Students Plans	Tutorial 9: Analyzing MD: Students Plans	How to prepare a barbecue
16:00	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break	
17:30	End of session	End of session	End of session	End of session	End of session	End of session	End of session	End of session	End of session	
20:30	Free time	Free time	Free time	Free time	Beer, Science & Friendship Good Stock Bar	Free time	Free time	Free time	Free time	Farewell

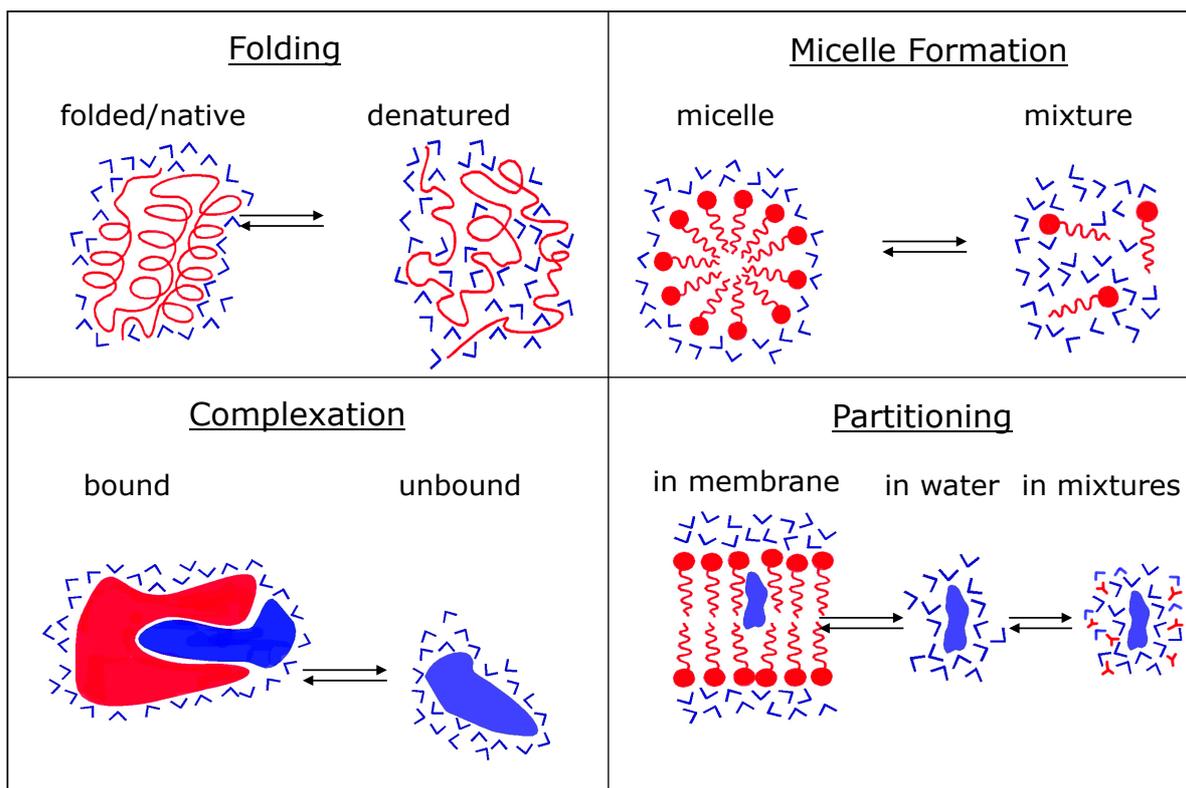
Outline

- **Free energies and drug design**
 - Enthalpy, Entropy and Free Energy
 - Actors and effects

- **Calculation of free energies from statistical mechanics**
 - Free energy perturbation
 - Thermodynamic integration
 - Slow / fast growth
 - One step perturbations

- **Conclusions**

Processes: Thermodynamic Equilibria



Definitions

free energy

The driving force for all physical processes
 Free energy ΔA ; Free enthalpy $\Delta G (= \Delta A + p\Delta V)$

energy

The internal energy of the systems
 Energy $\Delta E/\Delta U$; Enthalpy $\Delta H (= \Delta E + p\Delta V)$

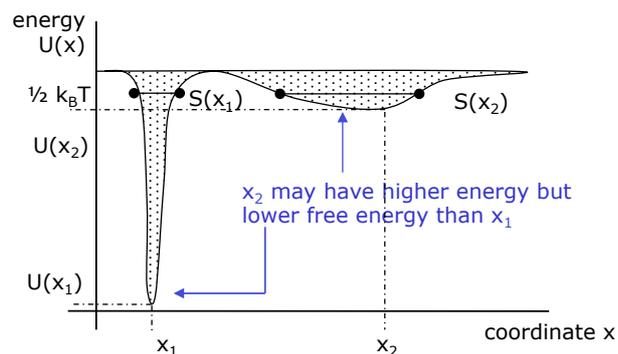
entropy

“The number of realization possibilities”
 Entropy ΔS

Helmholtz / Gibbs equations

$$\Delta A = \Delta E - T\Delta S; \quad \Delta G = \Delta H - T\Delta S$$

Statistical mechanics

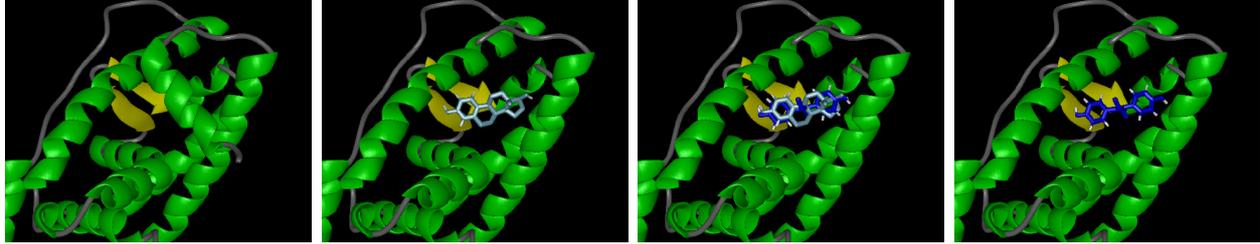


mechanics: a state is characterised by **one minimum energy structure** (global minimum)

statistical mechanics: a state is characterised by **an ensemble of structures** or configurations or conformations

Lock and key: binding affinity

- Predict binding affinity to prioritize compounds,
safe money on the experiments



The keyhole: the *active site* in a protein

A fitting key: the *active site* with a known binder

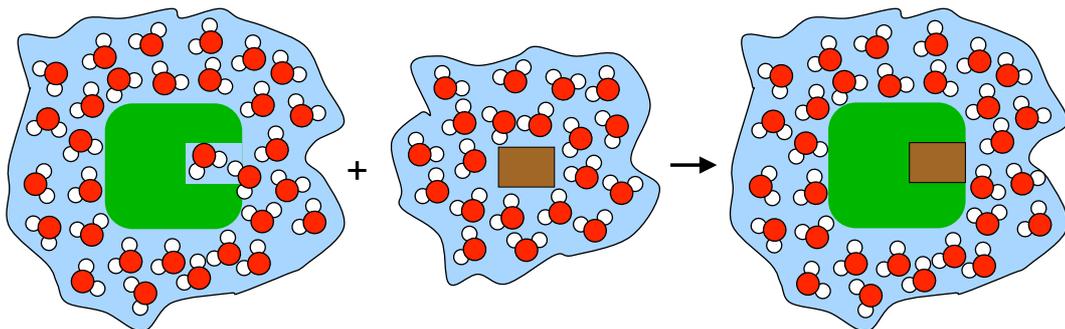
Old and new molecules superposed

A new key? The *active site* containing a new molecule

- Interaction between the **protein** and a **ligand**
- Affinity is determined by the **Free Energy**, (\neq potential energy)
- Free energy includes **entropy** (number of possible conformations, sampling!)



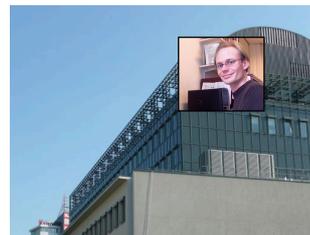
The system

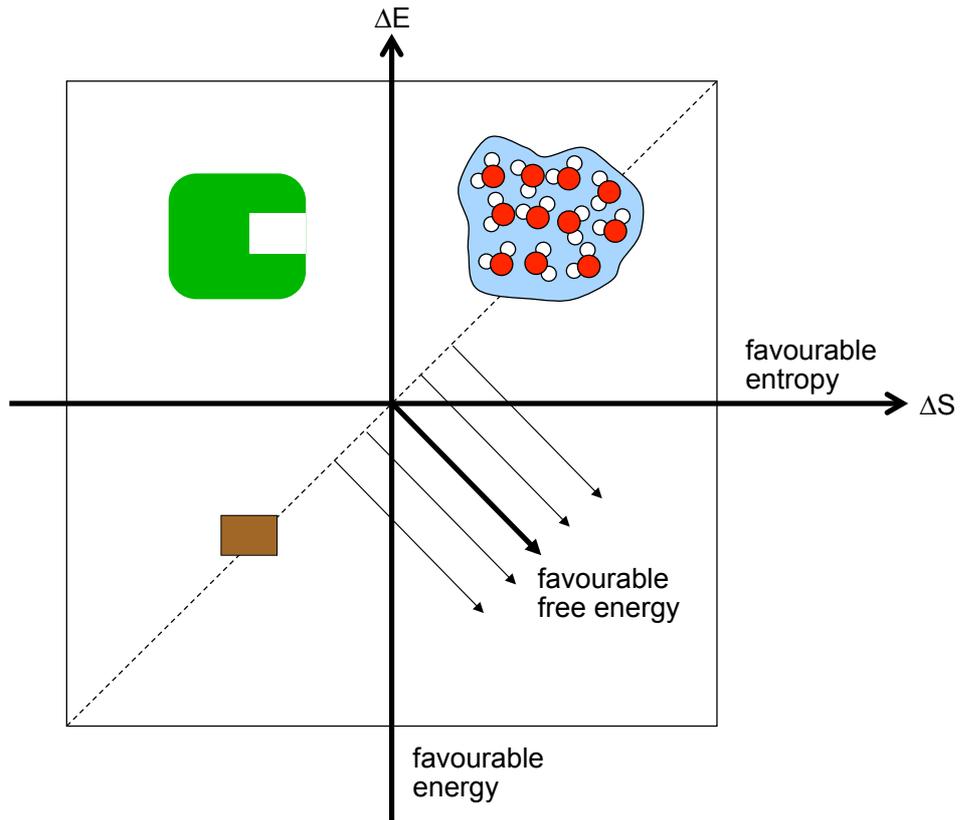


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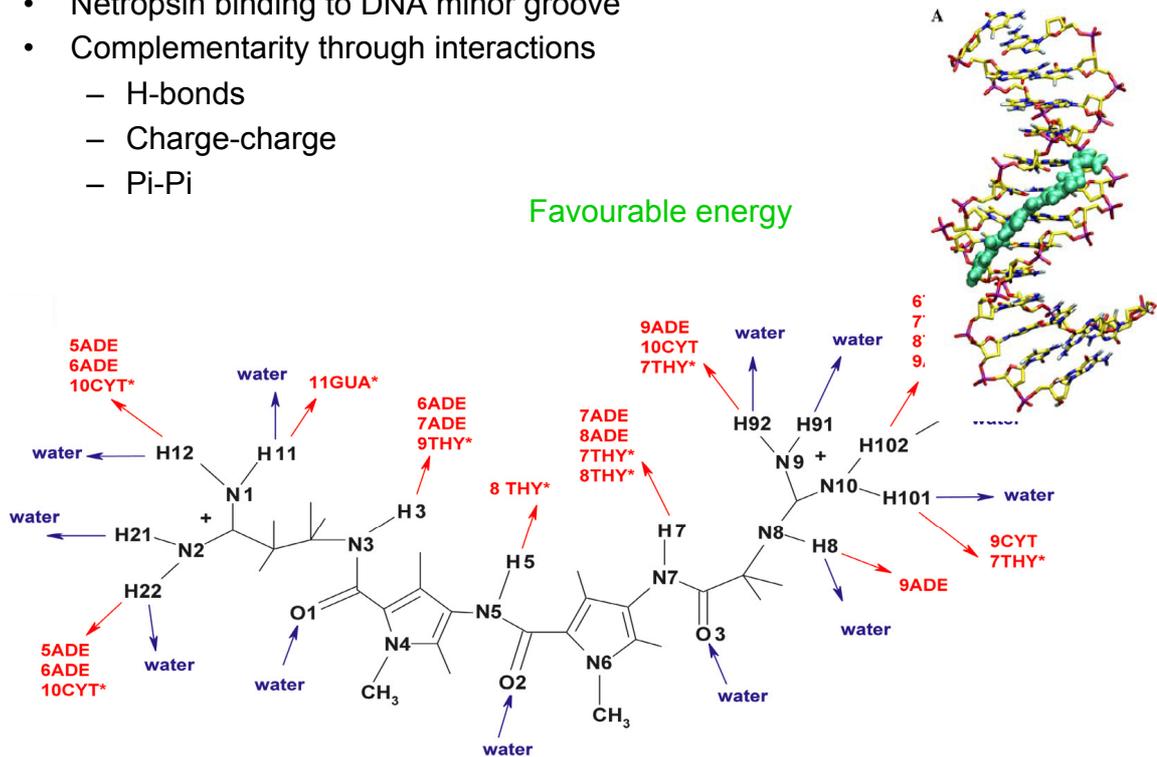




Guest-host complementarity

- Netropsin binding to DNA minor groove
- Complementarity through interactions
 - H-bonds
 - Charge-charge
 - Pi-Pi

Favourable energy



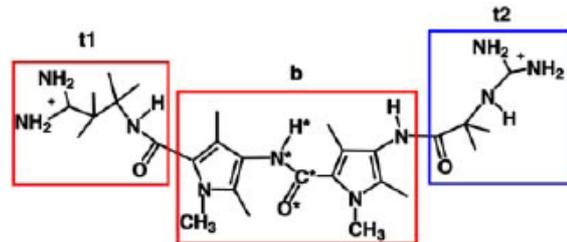


Entropy loss upon binding

- Conformational entropy calculated using Schlitter's formula
 - Netropsin and Distamycin A
 - In solution and when bound to DNA

	S_{free}	S_{bound}	ΔS_{bind}
Netropsin	862	735	-127
Distamycin	902	798	-104

in J/K/mol



Unfavourable entropy

Entropy loss mostly in the tails of the molecules

J. Dolenc, R. Baron, C. Oostenbrink, J. Koller and W.F. van Gunsteren, *Biophys J.* (2006) 91:1460

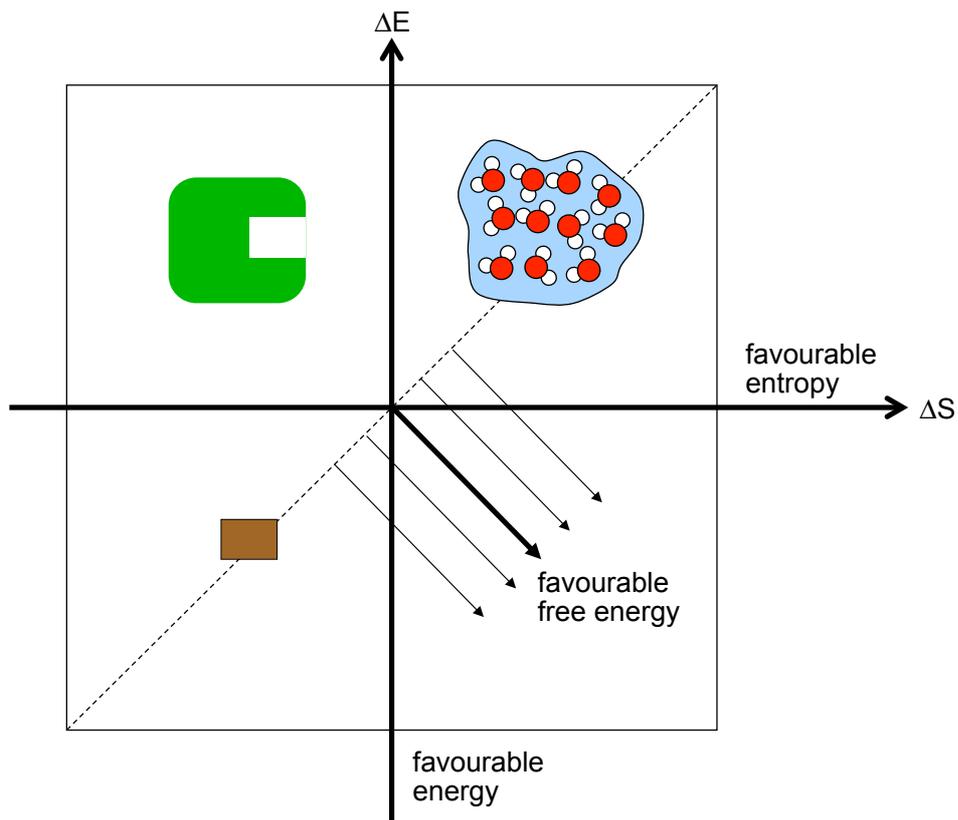


My affinity for BOKU

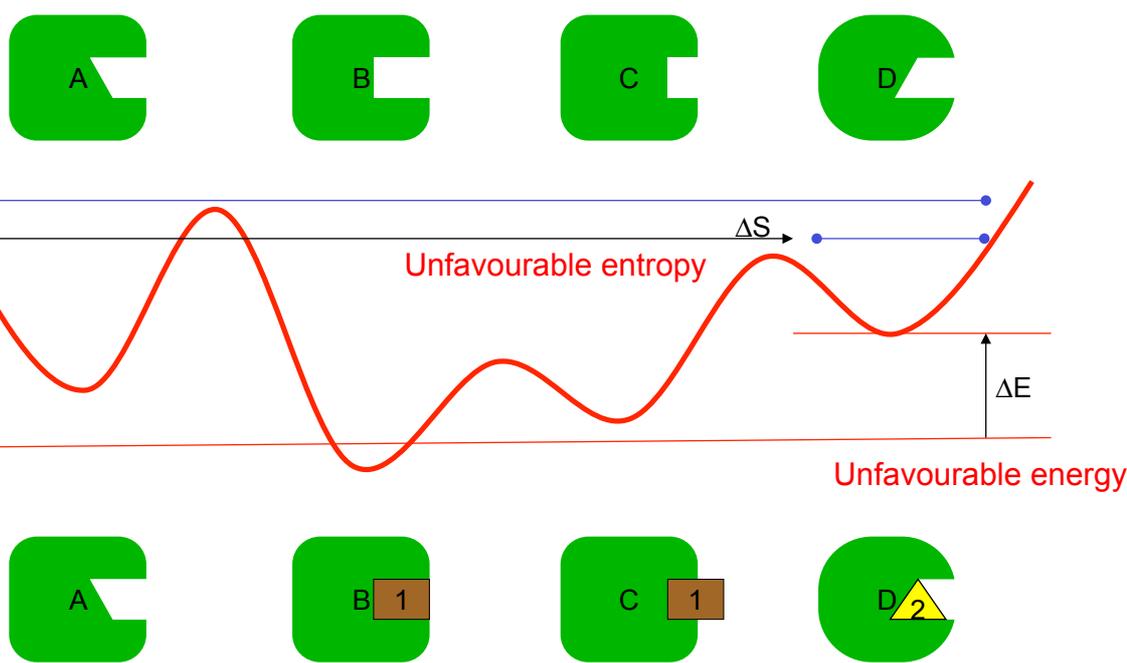
- Affinity is a combination of energy and entropy
 - Energy:
 - Interactions with people at BOKU are mostly favourable
(and I am even getting paid for it!)
 - Entropy:
 - How much freedom do I have? At BOKU and elsewhere?

Physical Map of the World, June 2003



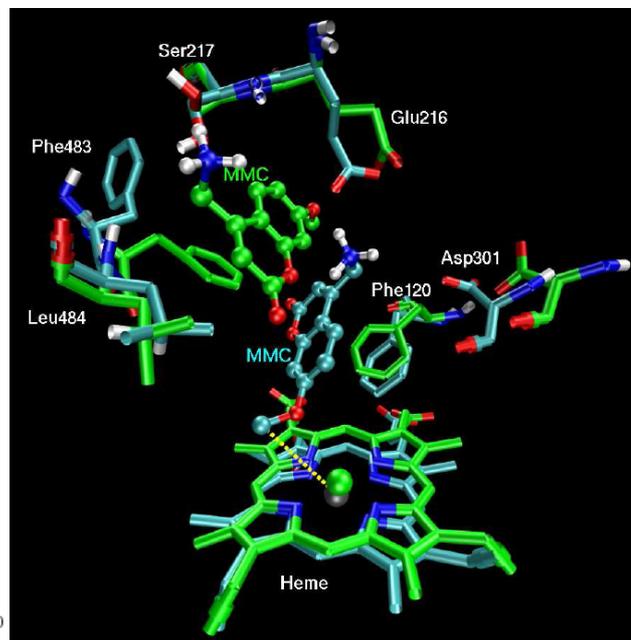
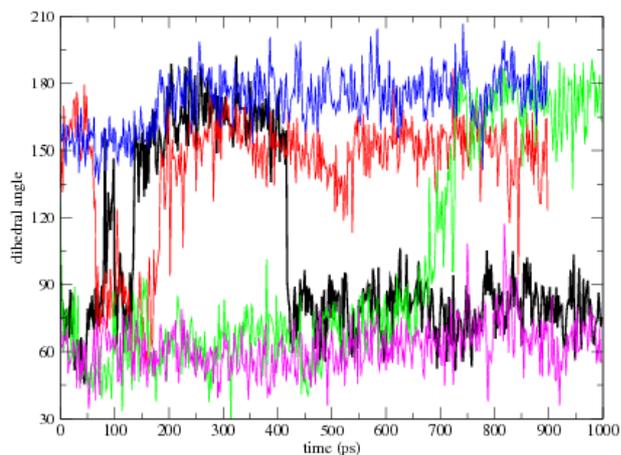


Conformational selection



Binding to multiple structures

- Docking of 65 substrates in 2500 protein CYP2D6 structures
- Side-chain of Phe483 occupies multiple conformational states
- Different substrates have different preference



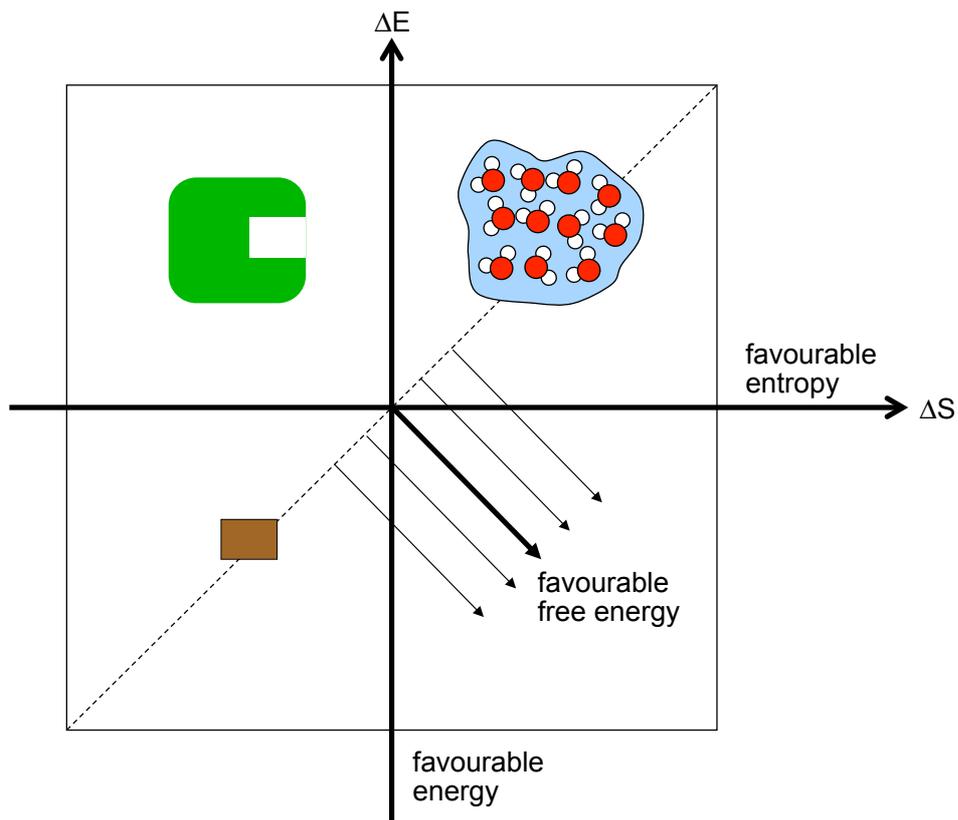
J. Med. Chem. 51 (2008) 7469 - 7477

The shape of the host

- Before we could come, people had to move and squeeze together
- Furniture was moved around

Unfavourable energy
Unfavourable entropy



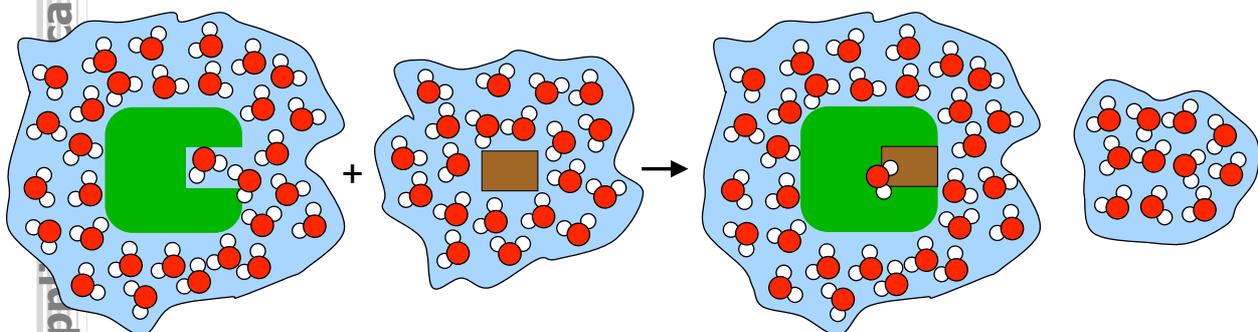


Desolvation of protein and ligand

- Loss of solute-solvent hydrogen bonds
- Release of ordered water to bulk
 - Hydrophobic effect
- Structural water molecules in the active site

Unfavourable energy

Favourable entropy



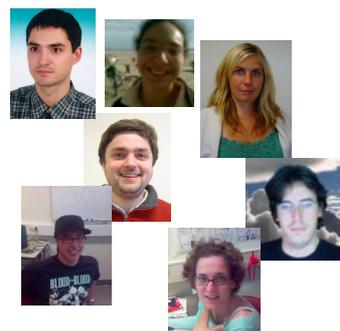
People and knowledge



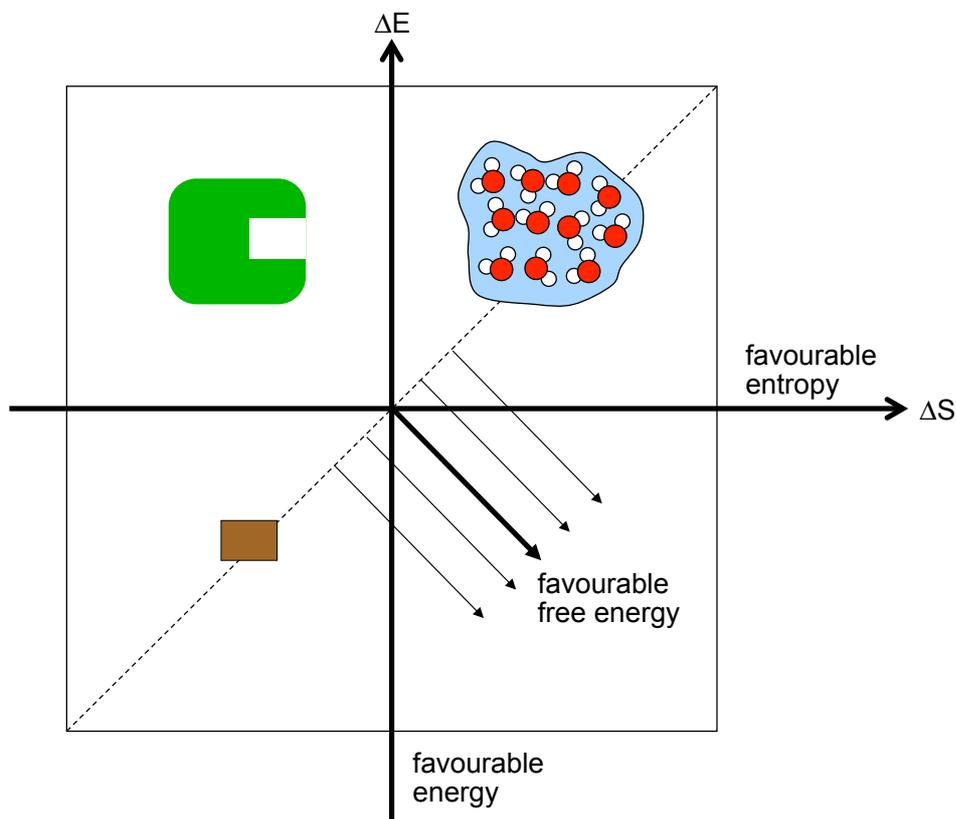
students and co-workers...



strengthen the interaction

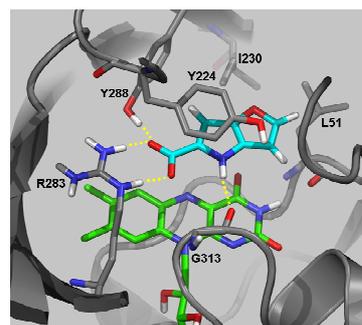
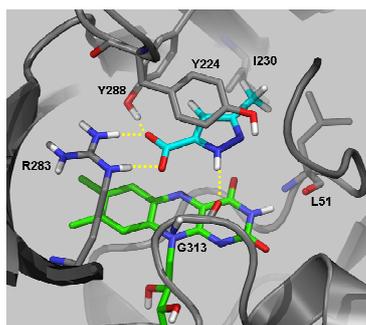
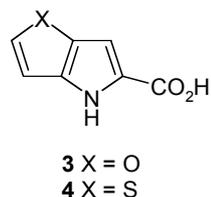
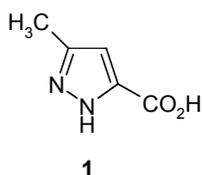


even if it hurts to say goodbye...
they have lot's of possibilities



Example: DAAO inhibitors

- Three inhibitors of the enzyme D-amino acid oxidase were studied



- Molecular dynamics simulations of the ligands in solution and bound to the protein, using GROMOS (parameter set 45A4)

Analysis of the simulations

Number of hydrogen bonds

	1	3	4
<i>Free in solution</i>	9.98	8.47	7.60
<i>In complex:</i>			
Tyr228 OH	1.00	1.00	1.00
Arg283 HE	1.03	0.99	1.00
Arg283 HH	1.28	1.35	1.34
Gly313	0.91	0.50	0.50
H ₂ O	2.15	1.87	0.61
Loss of H-bond	3.61	2.76	3.15

water is being released

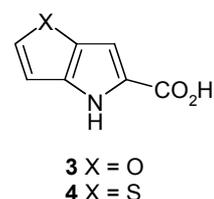
water still plays a role

Entropy (kJ/mol)

	calculated conformational entropy ligand	protein	full entropy experiment
-TΔS (3)	15.3	17.0	-5.0
-TΔS (4)	15.1	3.3	-15.9
-TΔΔS	0.2	13.7	10.9

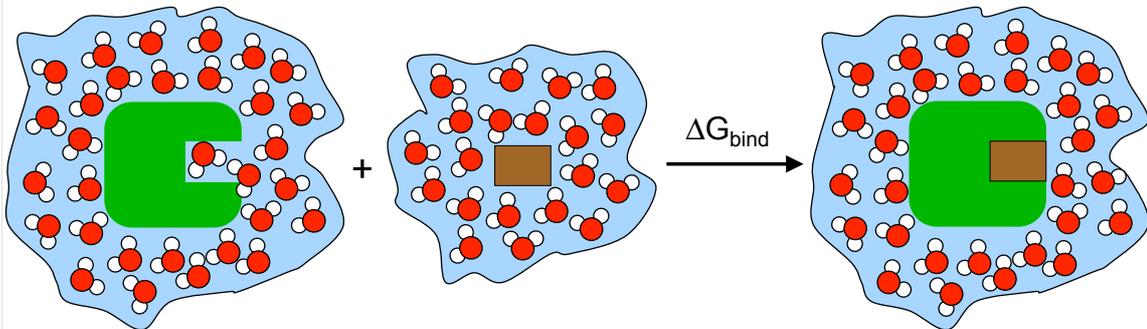
the ligands are equally rigid

protein loss of entropy explains experiment?



Free energies of binding

- We hope to calculate the free energy of ligand binding
 - And need to consider all energetic and entropic contributions

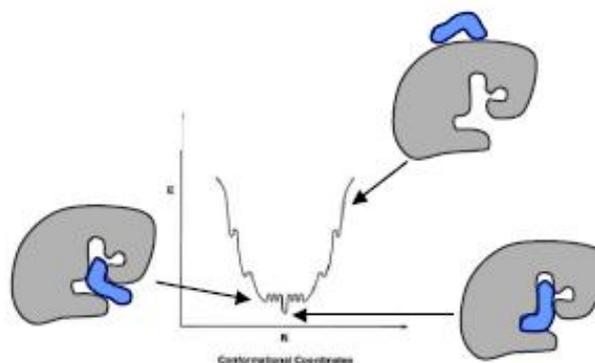


$$\Delta G_{bind} = G_{complex} - G_{ligand} - G_{protein}$$

Scoring problem

- Affinity is determined by free energy
- Needs to be calculated fast and accurately
- The details are on a small energy scale!

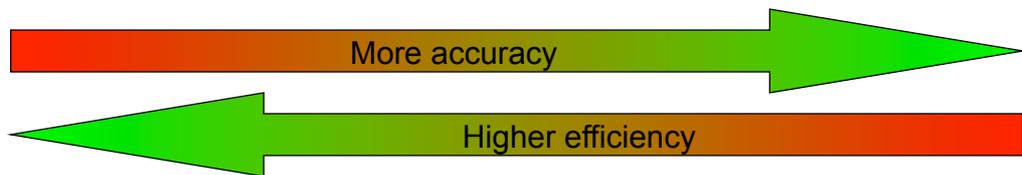
Goal: estimate binding affinity for given protein, ligand, pose, and conformations



Protein ligand interactions

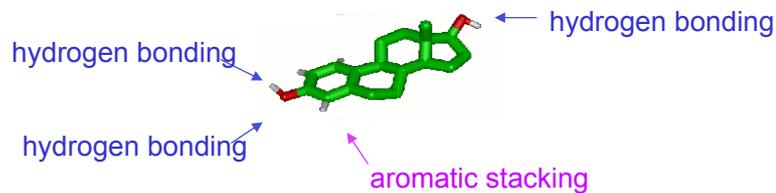
- Methods to calculate the interactions for different problems

	DOCKING: e.g. Genetic algorithm	LIE method: Molecular dynamics	ONE STEP: Molecular dynamics enhanced by soft atoms Statistical mechanics	FEP/TI: Molecular dynamics (extensive sampling) Statistical mechanics
Sampling				
Scoring	Empirical scoring function	Linear response		
Characteristics	Many empirical parameters Empirical function	Few empirical parameters Theoretical basis	No empirical parameters Theoretically correct Restricted to one framework	No empirical parameters Theoretically correct
	Rigid protein No solvation	Protein flexibility Solvation	Protein flexibility Solvation	Protein flexibility Solvation



Scoring functions

Estimate the binding free energy of a compound with a protein

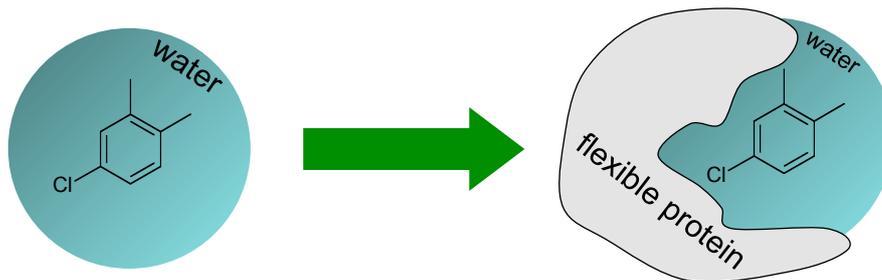


$$\Delta G^{\text{bind}} = \Delta G^{\text{HB}} + \Delta G^{\text{arom}} + \Delta G^{\text{hydr}} - T\Delta S^{\text{flex}} + \dots - \dots$$

Count the number of hydrogen bonds, aromatic interactions etc.

Entropy taken into account in a very rough (and wrong) way:
number of rotatable bonds, fuzzy factors to allow for protein flexibility

Linear interaction energies



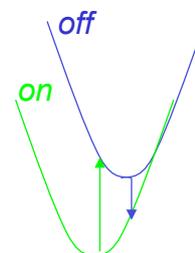
$$\Delta G_{bind} = \beta \left(\langle V_{l-s}^{el} \rangle_{bound} - \langle V_{l-s}^{el} \rangle_{free} \right) + \alpha \left(\langle V_{l-s}^{vdw} \rangle_{bound} - \langle V_{l-s}^{vdw} \rangle_{free} \right)$$

Derived from theory, $\beta=1/2$,
but $\langle V_{l-s} \rangle_{off} = 0$ assumed

No theoretical background,
empirical equation

- From linear response

$$\Delta G_{sol}^{el} = \frac{1}{2} \left(\langle \Delta V_{on \rightarrow off}^{el} \rangle_{on} - \langle \Delta V_{off \rightarrow on}^{el} \rangle_{off} \right)$$



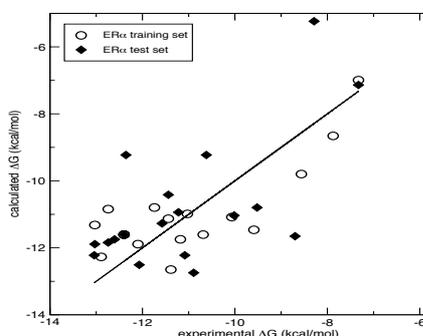
Direct estimates of ΔG_{bind}

- Can MD simulations be used in a lead optimisation project?
- Estrogen receptor a
 - 36 compounds, 18 training set, 18 test set
 - comparison of
 - Linear Interaction Method from MD simulation

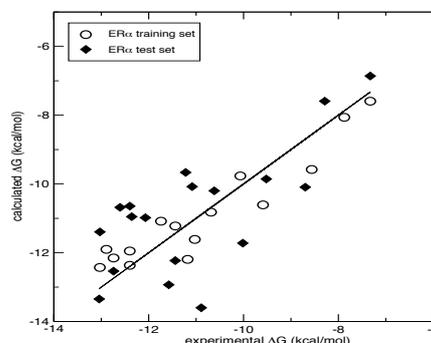
$$\Delta G_{bind} = \beta \Delta \langle V_{l-s}^{el} \rangle + \alpha \Delta \langle V_{l-s}^{vdw} \rangle + \delta \Delta \left(\langle V_{l-l}^{el} \rangle + \langle V_{l-l}^{vdw} \rangle \right)$$

- the best scoring function (selected out of 10) : HS score
linear regression from score to exp. binding free energies

LIE vs scoring?



HS score



LIE

r^2	train	0.62	0.87
rmsd	train	1.05 kcal/mol	0.63 kcal/mol
rmsd	test	1.54 kcal/mol	1.30 kcal/mol
Spearman-rank	all	0.55	0.65

Definitions

free energy

$$A(N, V, T) = -k_B T \ln \left[N! h^{3N} \right]^{-1} \iint \exp(-H(\vec{p}, \vec{r}) / k_B T) d\vec{p} d\vec{r}$$

$$= U - TS$$

energy

$$U(N, V, T) = \left(\frac{\partial A / T}{\partial 1 / T} \right)_{N, V} = \langle H(,) \rangle_{\vec{p}, \vec{r}}$$

entropy

$$S(N, V, T) = - \left(\frac{\partial A}{\partial T} \right)_{N, V} = \frac{U - A}{T}$$

Partition function

$$Z(N, V, T) = \left[N! h^{3N} \right]^{-1} \iint \exp(-H(\vec{p}, \vec{r}) / k_B T) d\vec{p} d\vec{r}$$

Free energy, energy and entropy are defined from statistical thermodynamics

Statistical mechanics

- Equation to calculate free energy is simple:

$$G = -k_B T \ln Z_{NpT}$$

- Where Z_{NpT} is the partition function of the system

$$Z_{NpT} = \frac{1}{h^{3N} N!} \iiint e^{-(H(\mathbf{r}, \mathbf{p}) + pV)/k_B T} d\mathbf{p} d\mathbf{r} dV$$

Simulation samples over positions, momenta and volumes in physically relevant way

Integral over **all** possible positions, and **all** possible momenta of **all** particles in all different volumes

Sampling of all positions

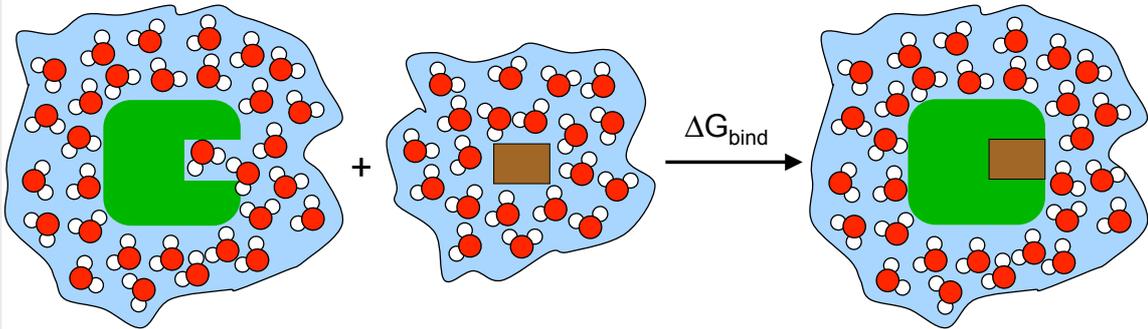
- Sampling of all positions to calculate absolute free energy is practically impossible
- For molecules and for me

Physical Map of the World, June 2003



Do we really want that? (I)

- We only really care about the difference, ΔG



- The perturbation formula calculates free energy differences directly

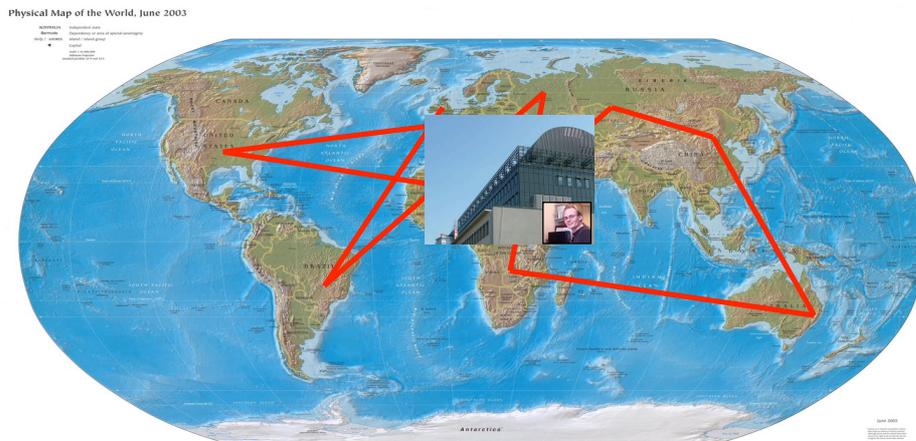
$$\Delta G = -k_B T \ln \left\langle e^{-(H_{\text{complex}} - H_{\text{ligand}} - H_{\text{protein}}) / k_B T} \right\rangle$$



Works if the positions, momenta and volumes of both sides are not too different

Directly consider the difference

- Consider my presence at BOKU as one of many possibilities

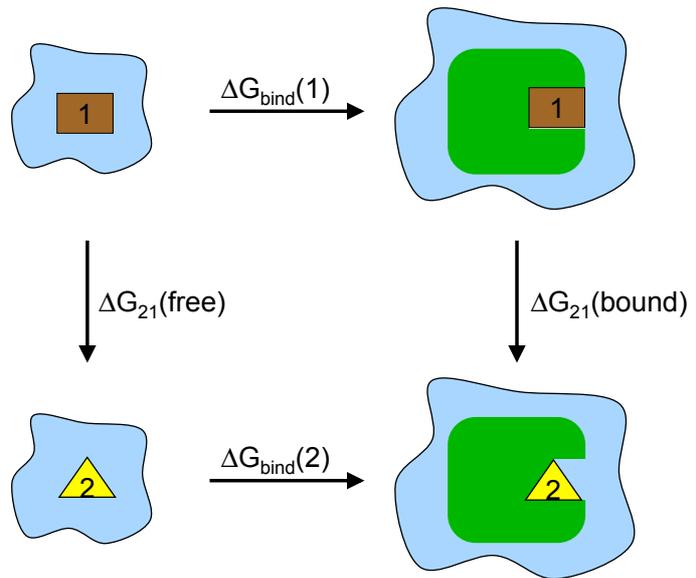


- Is still very difficult and unlikely to work



Do we really want that? (II)

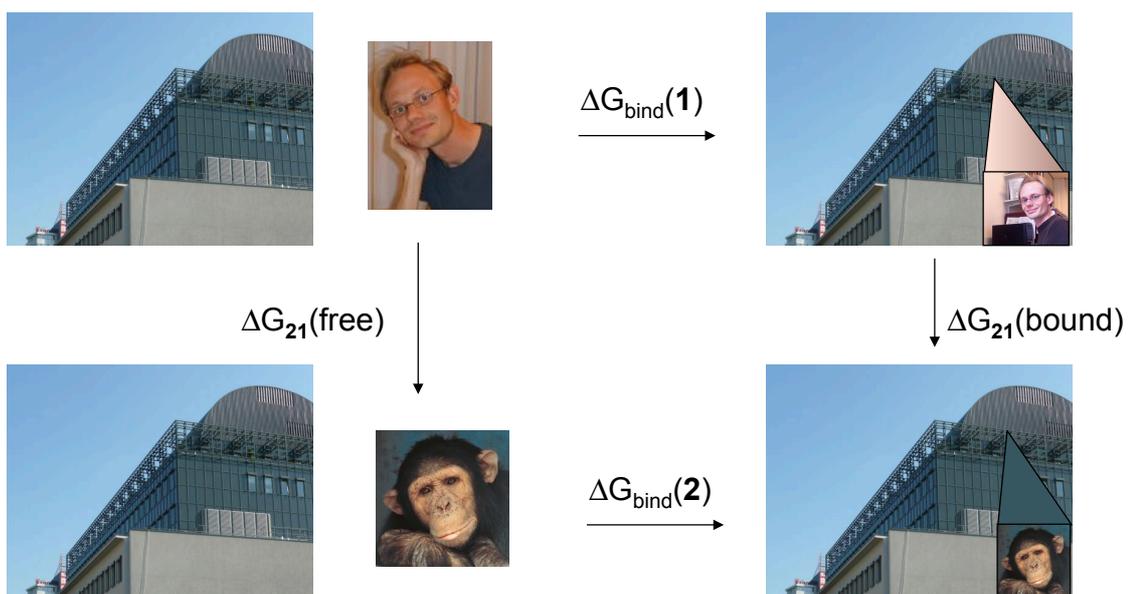
- Free energy is independent of the path (state function)
- Thermodynamic cycle
- Relative free energies
- Computational alchemy



$$\begin{aligned} \Delta\Delta G_{\text{bind}} &= \Delta G_{\text{bind}}(2) - \Delta G_{\text{bind}}(1) \\ &= \Delta G_{21}(\text{bound}) - \Delta G_{21}(\text{free}) \end{aligned}$$

Who fits better at BOKU?

- Are there others that are more suitable?



- Compare two employees when they are free and at BOKU

Free energy difference

Free energy perturbation

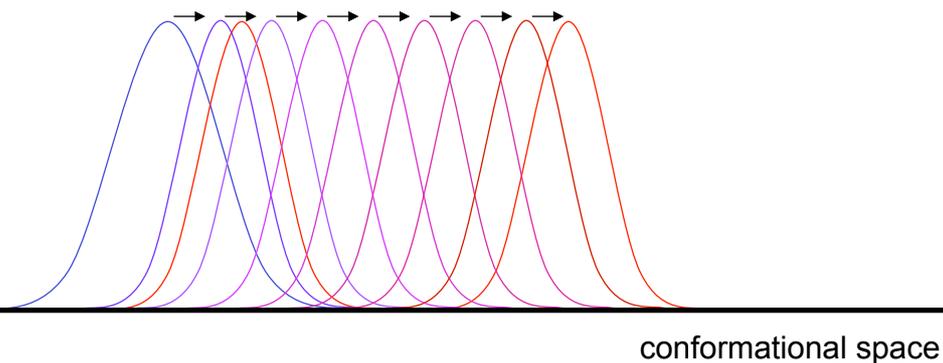
$$\begin{aligned}
 \Delta A_{BA} &= A_B - A_A = -k_B T \ln \frac{Z_B(N, V, T)}{Z_A(N, V, T)} \\
 &= -k_B T \ln \frac{\iint e^{-H_B(\mathbf{r}, \mathbf{p})/k_B T} d\mathbf{p} d\mathbf{r}}{\iint e^{-H_A(\mathbf{r}, \mathbf{p})/k_B T} d\mathbf{p} d\mathbf{r}} \\
 &= -k_B T \ln \frac{\iint e^{-(H_B(\mathbf{r}, \mathbf{p}) - H_A(\mathbf{r}, \mathbf{p}))/k_B T} e^{-H_A(\mathbf{r}, \mathbf{p})/k_B T} d\mathbf{p} d\mathbf{r}}{\iint e^{-H_A(\mathbf{r}, \mathbf{p})/k_B T} d\mathbf{p} d\mathbf{r}} \\
 &= -k_B T \ln \left\langle e^{-(H_B - H_A)/k_B T} \right\rangle_A
 \end{aligned}$$

Zwanzig, J. Chem. Phys (1954) 22:1420

Free energy calculation

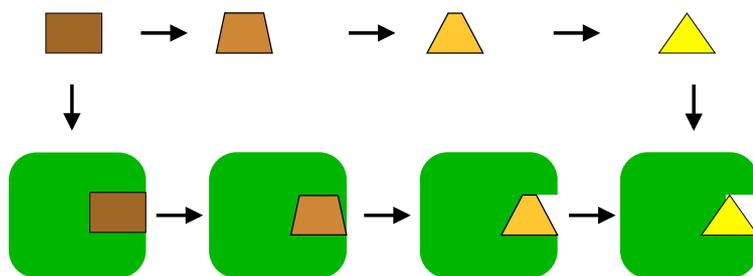
$$\Delta G_{AB} = -k_B T \ln \left\langle e^{-(E_A - E_B)/k_B T} \right\rangle_B$$

$$\Delta G_{AB} = \sum_{\lambda} \Delta G_{\lambda, \lambda + \delta\lambda}$$

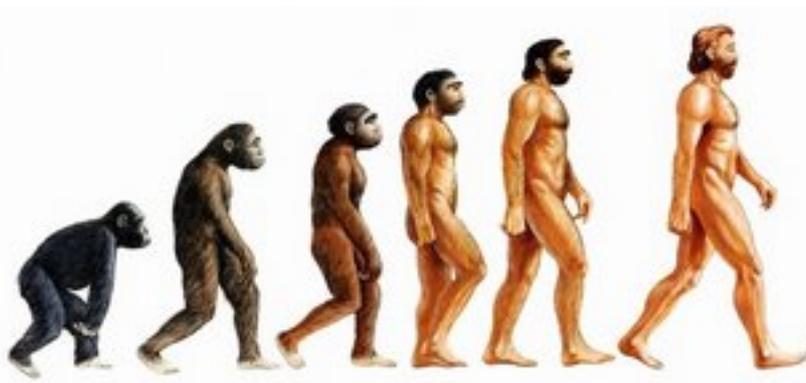


conformational space

Gradually change one in the other



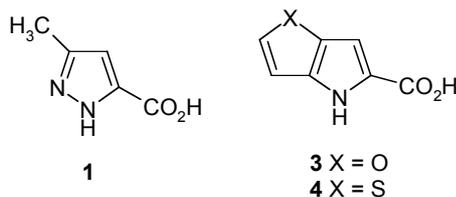
Change ligand 1 into ligand 2, in solution and when bound to the protein



As long as the end-states are defined, the intermediates do not have to be physically possible

Example: DAAO inhibitors

- Three inhibitors of the enzyme D-amino acid oxidase were studied

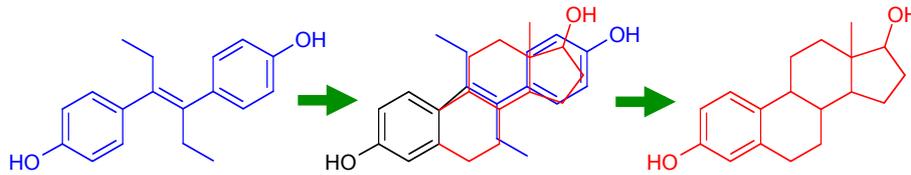


	3->1	3->4	4->1
<i>Calculated values:</i>			
ΔG_{free}	106.3 ± 1.5	86.1 ± 0.8	20.4 ± 1.1
$\Delta G_{\text{complex}}$	113.8 ± 2.2	87.3 ± 3.5	36.7 ± 2.0
$\Delta\Delta G_{\text{bind}}$	7.5 ± 3.7	1.2 ± 4.3	16.3 ± 3.1
<i>Experimental $\Delta\Delta G_{\text{bind}}$ based on:</i>			
IC ₅₀ ^a	8.2	-0.9	9.1
IC ₅₀ ^b	4.6	0.1	4.6
ITC	9.4	0.8	8.6
SPR ^c	14.1	1.6	12.4

Overall, the relative binding free energies are very well reproduced

Computational alchemy

- Modify one compound into another one in small steps



- In a formula: $E(\mathbf{q}, \mathbf{p}, \lambda) = (1 - \lambda)E_A(\mathbf{q}, \mathbf{p}) + \lambda E_B(\mathbf{q}, \mathbf{p})$

$$\lambda = 0 \rightarrow E = E_A \quad \lambda = 1 \rightarrow E = E_B$$

Along the way? The protein 'sees' a **mixture** of A and B

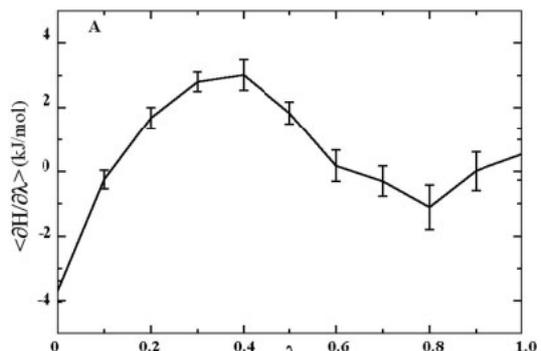
$$\Delta G_{AB} = \sum_{\lambda=0}^1 -k_B T \ln \left\langle e^{-\Delta E(\lambda \rightarrow \lambda + d\lambda) / k_B T} \right\rangle$$

Thermodynamic integration

- Slightly more convenient

$$\begin{aligned} \Delta A_{BA} &= \int_0^1 \frac{dA(\lambda)}{d\lambda} d\lambda \\ \frac{dA(\lambda)}{d\lambda} &= \frac{d}{d\lambda} \left(-k_B T \ln \frac{1}{h^{3N} N!} \iint e^{-H(\mathbf{r}, \mathbf{p}, \lambda) / k_B T} dp d\mathbf{r} \right) \\ &= -k_B T \frac{\frac{d}{d\lambda} \iint e^{-H(\mathbf{r}, \mathbf{p}, \lambda) / k_B T} dp d\mathbf{r}}{\iint e^{-H(\mathbf{r}, \mathbf{p}, \lambda) / k_B T} dp d\mathbf{r}} \\ &= \frac{\iint \frac{\partial H(\mathbf{r}, \mathbf{p}, \lambda)}{\partial \lambda} e^{-H(\mathbf{r}, \mathbf{p}, \lambda) / k_B T} dp d\mathbf{r}}{\iint e^{-H(\mathbf{r}, \mathbf{p}, \lambda) / k_B T} dp d\mathbf{r}} \\ &= \left\langle \frac{\partial H(\lambda)}{\partial \lambda} \right\rangle_{\lambda} \end{aligned}$$

Kirkwood, J. Chem. Phys (1935) 3:300



TI uses the derivative of the Hamiltonian to calculate free energies



Example: ER

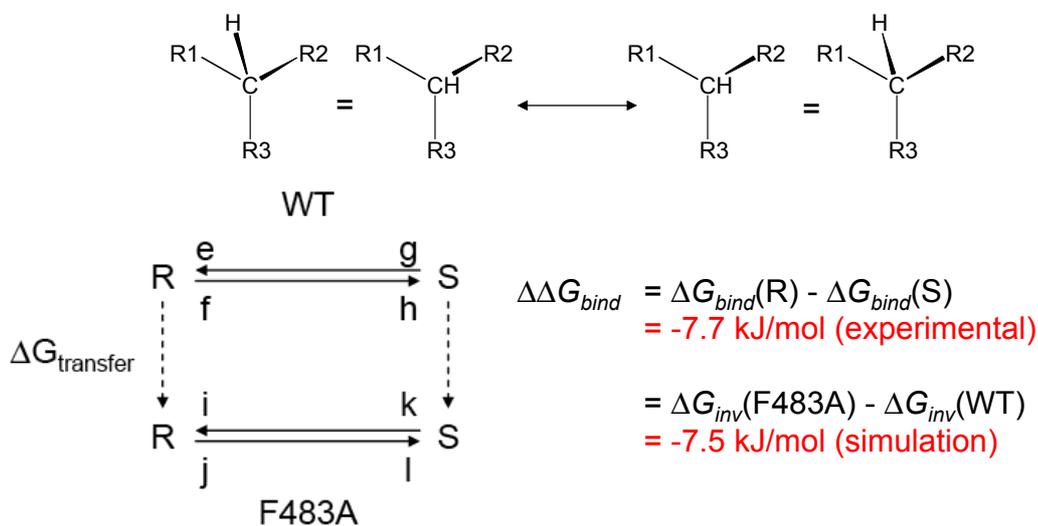
- Relative free energy of three compounds
- In three different media (vacuum, solution, protein)
- In 11 discrete steps, forward and backward TI

Table 4. TI Results (kJ mol⁻¹)^a

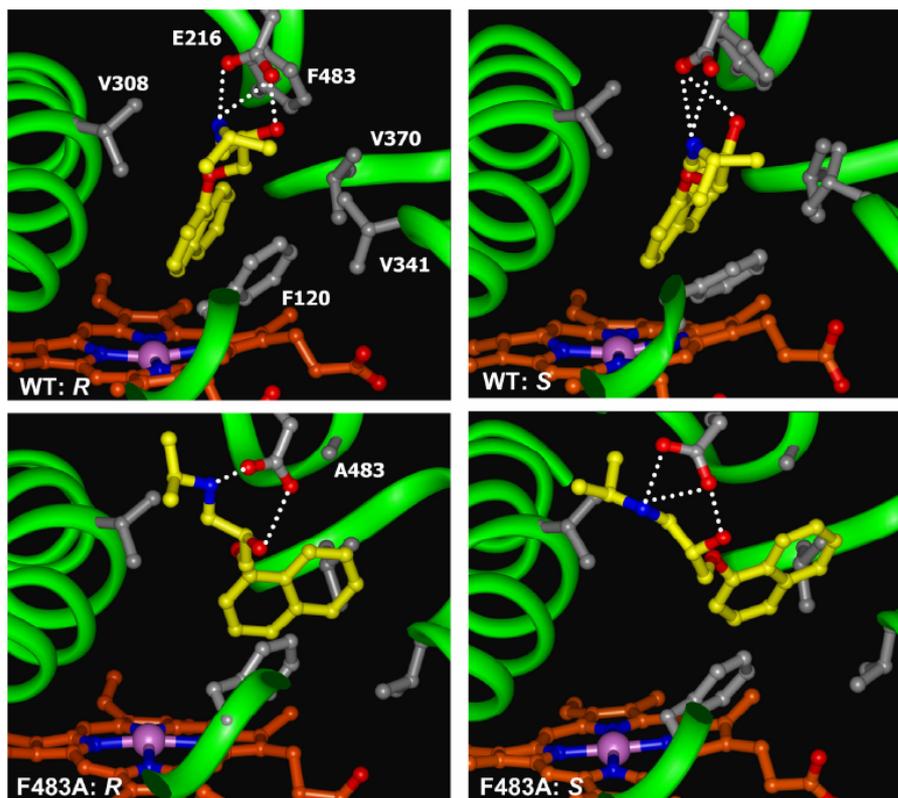
TI	DES ↔ E2			DES ↔ GEN		
	for-ward	back-ward	hysteresis	for-ward	back-ward	hysteresis
vacuum	76.3	76.1	0.2	187.1	186.9	0.2
solvent	79.0	81.6	-2.6	151.5	157.3	-5.8
protein	80.4	78.2	2.2	173.1	165.3	7.8
$\Delta\Delta G_{solv}$	2.8	5.5	-2.7	-35.6	-29.5	-6.0
$\Delta\Delta G_{bind}$	1.4	-3.4	4.8	21.6	8.0	13.6
$\Delta\Delta G_{bind}$ (expt)	3.8 ^b	0.79 ^c		11.3 ^b	21.69 ^c	

Stereospecific propranolol binding

- R- and S-Propranolol have similar affinity for CYP450 2D6
- 20 fold decrease of affinity of R-Propranolol to F483A mutant
- Free energy calculation to convert R-propranolol into S-propranolol



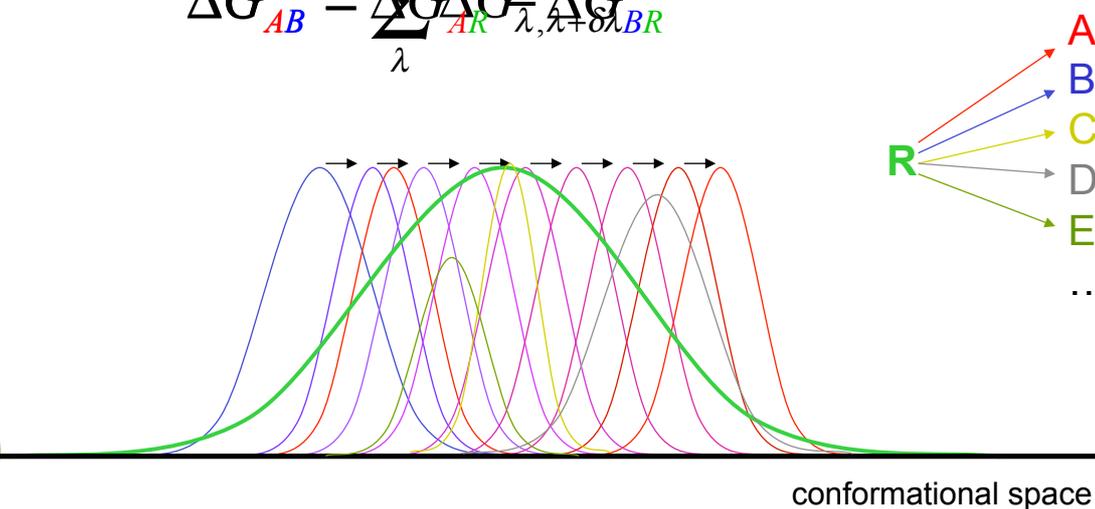
Molecular picture



One step perturbation Enveloping Distribution Sampling

$$\Delta G_{AB} = -k_B T \ln \left\langle e^{-(E_A - E_B)/k_B T} \right\rangle_B$$

$$\Delta G_{AB} = \sum_{\lambda} \Delta G_{AR} - \Delta G_{BR}$$

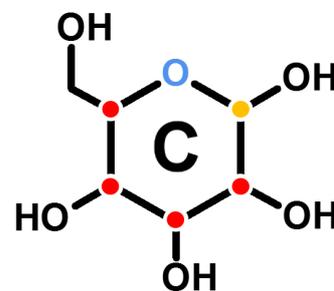
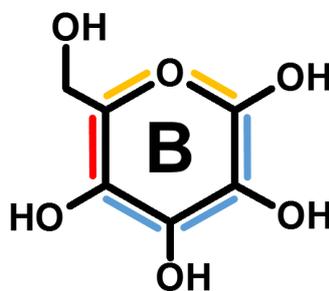
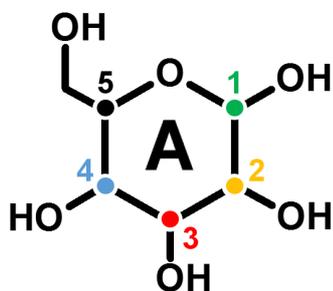
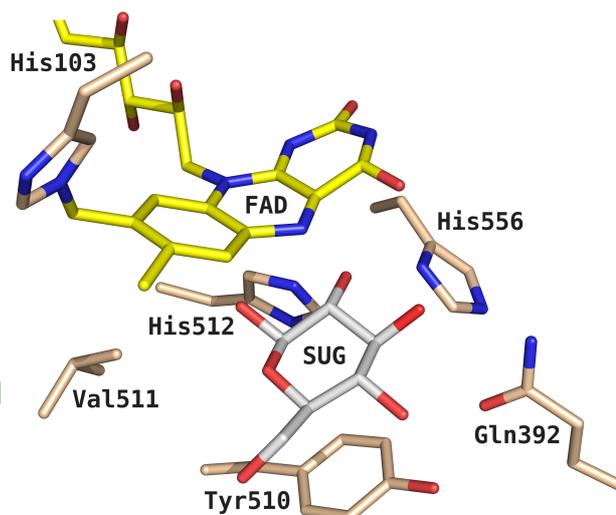


Pyranose dehydrogenase

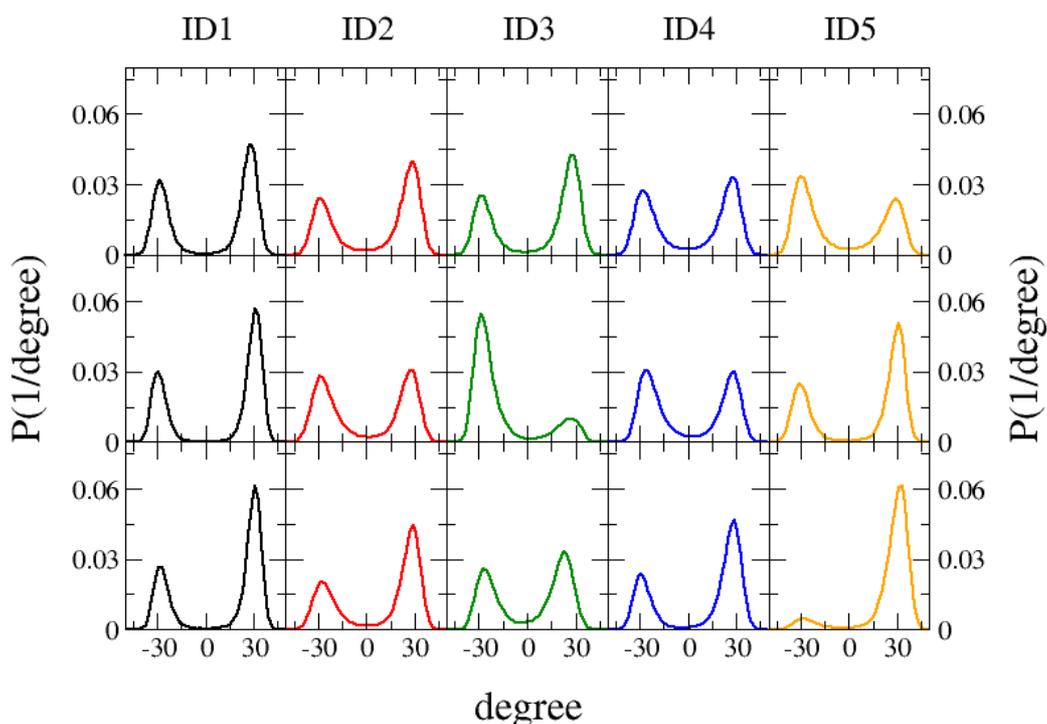
- fungal dehydrogenase
- litter degrading enzyme
- promiscuous substrate binding and product formation

- **Floppy reference states:**

- Improper dihedrals **removed**
- + proper dihedrals **weakened**
- + angle bending **weakened**



Occurrences



- water one simulation; in protein multiple reference states used

Free energies

Sugar	ΔG_{water}	ΔG_{prot}	$\Delta\Delta G_{\text{bind}}$ [kJ/mol]
β -D-glucose	31.7	22.5	-9.1
β -D-mannose	37.9	46.7	8.7
β -D-galactose	34.5	29.2	-5.3
β -D-talose	40.8	41.7	0.8
α -D-glucose	34.4	31.5	-2.9
α -D-mannose	33.9	35.9	2.1
...			
β -D-gulose	27.6	26.6	-1.0
β -D-idose	32.7	40.0	7.3
α -D-allose	31.6	35.8	4.2
α -D-altrose	29.7	38.2	8.6
α -D-gulose	31.3	36.0	4.7
α -D-idose	32.6	36.3	3.7
β -L-glucose	30.8	40.1	9.3
β -L-mannose	30.1	35.3	5.2
β -L-galactose	28.0	44.3	16.3
β -L-talose	30.4	29.9	-0.5
α -L-glucose	31.2	62.0	30.8
α -L-mannose	27.2	39.1	12.0

Sugar	$\Delta G_{\beta-\alpha}$ exp.	$\Delta G_{\beta-\alpha}$ calc.
D-glucose	-1.2	-2.8
D-mannose	1.8	4.1
D-galactose	-1.7	-1.7
D-talose	1.0	3.0
D-allose	-4.1	-4.7
D-altrose	-1.1	-1.0
D-gulose	-4.8	-3.7
D-idose	-0.3	0.1

Sugar	$\Delta\Delta G_{\text{bind}}$ exp.	$\Delta\Delta G_{\text{bind}}$ calc.
D-glucose	0.0	0.0
D-mannose	12.3	10.9
D-galactose	0.6	3.7
D-talose	11.5	5.6

Conclusions

- Free energy calculations for e.g. drug design / lead optimisation
 - Enthalpic and entropic effects should be included
 - Statistical mechanics and thermodynamic cycles
 - Efficient calculations
 - Unphysical intermediates
- Different methods available
 - Docking and scoring
 - Empirical free-energy methods
 - Free energy perturbation
 - Thermodynamic integration
 - One-step perturbation from a specialized reference state
- Protein flexibility and multiple binding conformations contribute
- Accuracy ~ 2.5 kJ/mol